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Multi-objective optimization of the tooth surface in helical gears using design of experiment and the response surface method †

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Abstract

The design of tooth surface for low gear noise under various load conditions is very important, and gear noise is highly related to transmission error. Optimal tooth surface for reduction of transmission error is very difficult to analytically determine due to nonlinearity of transmission error and the need to satisfy multiple load conditions. Satisfying design variables in multiple load conditions leads to the Pareto optimum of multi-objective optimization. There, the method to determine optimal lead curve and robust tooth surface design is proposed, using the response surface method and multi-objective optimization. The effect of transmission error on the candidate design variables by a screening experiment has been investigated using analysis of variance. Design variables are likewise selected. The fitted regression model of transmission error is built with the statistic validation of the representation. The model with constraints is solved to obtain optimum lead curve design and robust design for the tooth surface under multiple loads.

Keywords: Transmission error; Helical gear; Design of experiment; Response surface method; Multi-objective optimization

1. Introduction

Gear design determines macro-geometry of gears through module, number of teeth, pressure angle, and other parameters. Micro-geometry such as tooth surfaces is likewise identified. Gear macro-geometry can be changed only during the first step of the design process; however, tooth surface micro-geometry can be modified even in the final step. As tooth surfaces are very sensitive to noise, their design has been recognized as an important tool for gear noise reduction. Tooth surface consists of the profile curve in the whole depth direction and the lead curve in the face width direction. The profile curve is important in most spur gears, while both the profile and lead curves are important in helical gears. Although gears do not have manufacturing errors, loaded gears have different contacts compared to unloaded gears due to twisting and bending moments. Such bending deflection causes the motion of gears to non-conjugate action. The profile curve should be modified so that gears recover conjugate action. Profile modifications such as tip-relief compensate for tooth-bending deflection. Gears also come into partial contact because of shaft misalignment, bearing clearance, and shaft and housing deformation, producing local stress and noise. Therefore, the original lead curve should be modified so that the gears do not come into partial contact, reducing noise and vibration. Lead modifications such as lead crowning compensate for manufacturing lead errors, shaft misalignment, and shaft deflection. Gear noise is highly related to transmission errors caused by gear deformation and tooth errors [1]. For gear design, optimal design of the tooth surface based on profile and lead modifications is important because small modifications can create considerable effect on noise.

As to reports on the optimum design of the tooth surface for helical gears, Conry and Seireg [2] proposed optimal tooth surface modification to minimize load distribution in the tooth contact zone. Maruyama et al. [3] dealt with the effects of direction of lead error on transmission error and bending moment of automobile transmission gears by the finite element analysis. The direction of lead error has little effect on the maximum bending moment; however, it has a great effect on transmission error, especially for the case with tooth contact to the leading side. Sundaresan et al. [4] dealt with the design of optimum gear tooth modifications that minimize transmission error and at the same time, are less sensitive to manufacturing errors, misalignment, and torque variances. They used statistical design of experiment concepts to minimize the effect of manufacturing and operational errors on transmission error, as an optimization procedure. Umeyama [5] proposed formulas

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of the transmission error of a helical gear pair, which consider the actual point of contact and misalignment of shafts, assuming that teeth are rigid. Umeyama et al. [6] also introduced the actual and effective contact ratios, and studied the effects of gear dimensions and modified tooth surfaces on the loaded transmission error. Regalado [7] dealt with the application of design of experiments and the Taguchi methods in the robust optimization of cylindrical gears with non-standard center distance based on a multi-objective criterion. The objectives for consideration are balance in pitting and bending life, transmission error, efficiency, and volume. Harianto and Houser [8] evaluated the effect of micro-geometry variation on noise excitations, gear contact and root stresses, film thickness, and surface temperature under loaded conditions by analytical simulations. Micro-geometry considers profile crowning, profile slope, lead crowning, lead slope, and bias modification variations. Previous studies have shown that various object functions such as load distribution, stress, and transmission error apply to macro-geometry and/or micro-geometry, by various methods. Previous studies have also needed much computational effort to obtain optimization.

Optimum design of the tooth surface for reduction of transmission error is very difficult to analytically determine due to nonlinearity of transmission error. However, if the fitted regression model of transmission error can be found, an easy approach to optimization can be obtained. Furthermore, the tooth surface should be designed to produce relatively low transmission error in all load conditions of operating load range. This leads to the Pareto solution of multi-objective optimization, which means a trade-off of the solution. This work focuses on the optimization of the tooth surface in helical gear for noise reduction by the fitted regression model of the transmission error. The following section explains the method to calculate transmission error, and develops the fitted regression model for optimization of lead curve by the design of experiment and the response surface method. If the model is statistically proven effective, it is used to design the optimal lead curve in the helical gears. Finally, robust design of the tooth surface with tip-relief and lead crowning for low transmission error under multiple load conditions is conducted using multi-objective optimization.

2. Transmission error analysis

Transmission error results from tooth deformation and error. Tooth deformation consists of bending deflection and contact deformation. This section describes procedures for calculating transmission error. Bending deflection, contact deformation, and modeling of the tooth surface are first mentioned, after which transmission error is calculated [9].

2.1 Bending deflection

Bending deflection is calculated by its influence function. The influence function of deflection uses the following approximate equation, as suggested by Umezawa, in the coordinate system (x, y, ξ,η) (Fig. 1) [10].

$$K_b(x,y,\xi,\eta) = U\frac{v(r)}{\sqrt{F(|x-\xi|)}\sqrt{G(|y-\eta|)}} \times \sqrt{F(x)}\sqrt{F(\xi)}\sqrt{G(y)}\sqrt{G(\eta)} \quad (1)$$

Here, G and F are the common characteristics of deflection in the whole depth direction and the face width direction, respectively. U is deflection at the tip. v is the common function of deflection. (x, y) is the coordinate of the measuring point. (ξ,η) is the coordinate of the loading point. The values of G, F, U, and v are obtained from bending deflection of finite element analysis by Nastran. The finite element model uses standard involute gear with the tooth foundation, as shown in Fig. 2. The horizontal length of the tooth foundation is six times the whole depth, and the vertical distance is three times the whole depth. The model uses parabolic solid elements and material properties of steel (Young's modulus =206 GPa, Poisson's ratio=0.3), and fixes the entire bottom and all sides of the tooth foundation in the x, y, z direction.

2.2 Contact deformation

Contact deformation has considerable contribution to deflection, as well as bending deflection. It has nonlinear characteristics wherein both deformation and contact width are

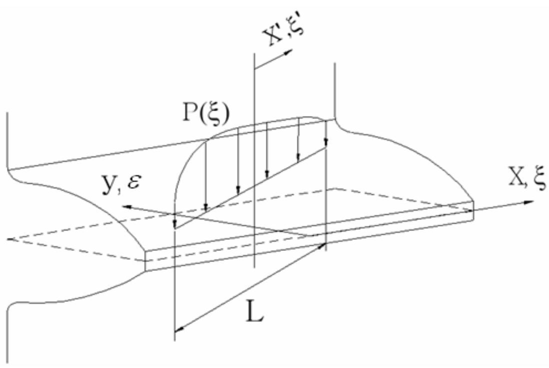


Fig. 1. Coordinate system.

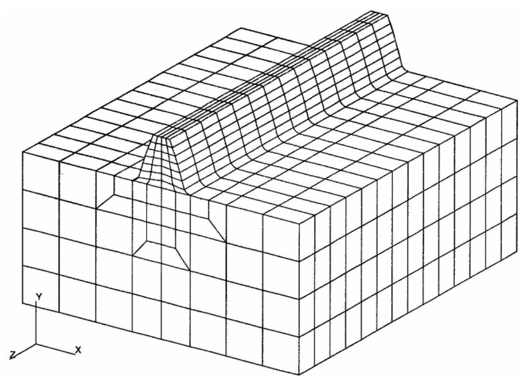


Fig. 2. Finite element model.

changed by the load. In this study, the influence function of contact deformation is used. To obtain the influence function of contact deformation, contact forces along the contact line are calculated under the constant contact deformation v_o, using the equation proposed by Weber [11]. The influence function of contact deformation $K_c(x)$ is given by

$$K_c(x) = \frac{v_o}{p_c(x)} . \quad (2)$$

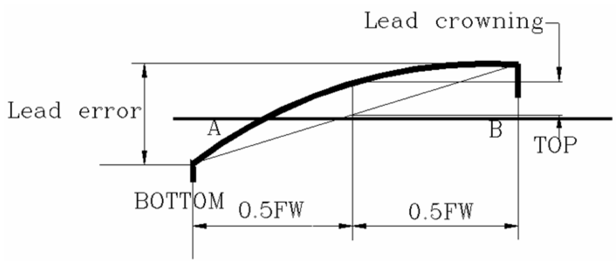
2.3 Modeling of the tooth surface

Modeling of the tooth surface is used to model the lead and profile curves. The lead curve consists of lead error and lead crowning, as shown in Fig. 3(a). Lead error is defined as a linear equation from the bottom of the lead curve to the top of the lead curve. Lead crowning is a quadratic equation from the bottom of the lead curve to the top of the lead curve based on the input data at the center of the face width. The profile consists of tip (root) relief, pressure angle error, and profile crowning, as shown in Fig. 3(b). Tip relief is modeled as a linear equation in the extent of the tip modification, and pressure angle error is defined as a linear equation from the root of the profile to the position starting the tip modification.

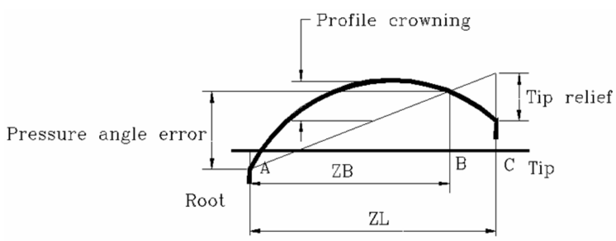
Profile crowning is defined as a quadratic equation from the root of the profile to the position starting the tip modification based on the input data at the center of the profile.

2.4 Calculation of transmission error

Transmission error Δ is calculated by bending deflection, contact deformation, and tooth error. The load deformation equation in the coordinate system (x', ξ') (Fig. 1) along the contact line is given by



(a) Lead curve



(b) Profile curve

Fig. 3. Modeling of the tooth surface.

$$\Delta = \int_{-L/2}^{L/2} K_b(x', \xi') p(\xi') d\xi' + K_c(x') P(x') + e(x') . \quad (3)$$

Here, $K_b(x', \xi')$ and $K_c(x')$ are obtained by the coordinate transform of Eqs. 1 and 2, respectively. $e(x')$ is the composite tooth errors that total tooth errors of a driving gear and a driven gear at the point x' of the contact line. The transmitted load W_j of tooth pair *j* is given by

$$W_j = \int_{-L/2}^{L/2} p(\xi') d\xi' . \quad (4)$$

The condition that the input torque divided by the base radius of the driving gear is the sum of the transmitted load of each meshing tooth pair leads to the following equation:

$$T / R_{b1} = \sum_j^n W_j \cos \beta_b . \quad (5)$$

The initial transmission error is assumed to calculate transmission error. By assuming transmission error, the nonlinear equation (Eq. (3)) is solved to obtain the tooth load distribution of $p(x')$. If the tooth load distribution does not satisfy Eqs. (4) and (5), the assumed transmission error is changed, and the same procedure is repeated to obtain the desired error range [12].

3. Optimization of the lead curve

The first interest of this work is the optimal design of the lead curve. Using the calculated transmission error, design of experiment, and response surface method, this section proposes the method to determine the optimum lead curve [13]. The object function for this purpose is the transmission error. Gear specification used in this analysis is shown in Table 1.

Lead error, lead crowning, and input torque are selected as the candidate design variables. To eliminate unimportant variables, the effect of transmission error analysis on the candidate ones for a screening experiment is investigated using analysis of variance. Table 2 shows the analysis of variance for lead

Table 1. Gear specification.

	Pinion	Gear
Normal module	2.5	
Normal pressure angle (deg)	20°	
Center distance (mm)	150	
Effective face width (mm)	48	
Helix angle (deg)	25°	
Number of teeth	19	89
Outside diameter (mm)	59.0965	250.9482
Pitch diameter (mm)	52.4105	245.5016
Root diameter (mm)	47.8914	239.7430
Addendum mod. co.	0.3372	0.0893

Table 2 .Analysis of variance for lead errors.

Source	F_o	P-value
Torque	2.7809	0.1211
CR1	0.3406	0.7212
CR2	0.3406	0.7212
Torque*CR1	1.9173	0.2010
Torque*CR2	1.9173	0.2010
CR1*CR2	1.8632	0.2104

Table 3. Analysis of variance for lead crownings.

Source	F_o	P-value
Torque	18.0183	0.0011
CR1	9.0300	0.0089
CR2	9.0300	0.0089
Torque*CR1	4.7213	0.0299
Torque*CR2	4.7213	0.0299
CR1*CR2	0.2160	0.9222

errors. The region of exploration is that torque is from 5 kgf·m to 15 kgf·m; lead error of the driving gear, LE1 in Table 2, is from 0 μm to 10 μm; lead error of the driven gear, LE2 in Table 2, is from 0 μm to 10 μm. Since P-values of LE1 and LE2 are very large, the lead error has no great effect on transmission error. This shows the same result as Harianto and Houser [8], where profile and lead slope corrections have little effect on transmission error. Table 3 shows the analysis of variance for lead crownings. The region of exploration is that torque is from 5 kgf·m to 15 kgf·m. Lead crowning of the driving gear, CR1 in Table 3, is from 7 μm to 21 μm. Lead crowning of the driven gear, CR2 in Table 3, is from 7 μm to 21 μm. Since P-values of torque, CR1, and CR2 approach zero, torque and lead crowning are significant on transmission error. Torque*LE1 and Torque*LE2 in Table 2, and Torque*CR1 and Torque*CR2 in Table 3, give the information of the interaction between torque and lead error and the interaction between torque and lead crowning, respectively. The interaction between sources can be similarly analyzed with torque, lead error, and lead crowning. The interaction between torque and lead crowning is strong, while interaction between torque and lead errors is weak.

As the result of the analysis of variance, the selected design variables are torque, lead crowning for a driving gear, and lead crowning for a driven gear. After the design variables have been selected at the region of the optimum, the second-order model for transmission error is used as follows:

$$y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j}^{k} \beta_{ij} x_i x_j + \varepsilon \ . \tag{6}$$

The central composite design for fitting the second-order model is introduced. The central composite design consists of a 2^k factorial, 2 k axial runs, and n_c center runs. The model can be written in matrix notation as

Table 4. Central composite design and experimental results (α=1).

Run	x_1	x_2	x_3	y
1	-1	-1	-1	0.2345
2	1	-1	-1	0.3656
3	-1	1	-1	0.6659
4	1	1	-1	0.3804
5	-1	-1	1	0.6659
6	1	-1	1	0.3804
7	-1	1	1	1.135
8	1	1	1	0.3627
9	-α	0	0	0.6659
10	α	0	0	0.3804
11	0	-α	0	0.1618
12	0	α	0	0.5841
13	0	0	-α	0.1618
14	0	0	-α	0.5841
15	0	0	0	0.3851

Table 5. Analysis of variance for significance of regression in multiple regressions.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_o	F(3,11;0.01)/ P-value
Regression	0.8501	3	0.28337	96.71	6.22/0
Error or residual	0.0322	11	0.00293		
Total	0.8823	14			

$$y = X\beta + \varepsilon \ . \tag{7}$$

The least squares estimator of β is then given by

$$\hat{\beta} = (X^T X)^{-1} X^T y \ . \tag{8}$$

The region of exploration to fit the second-order model is that torque is from 5 kgf·m to 15 kgf·m; lead crowning for the driving gear is from 7 μm to 21 μm; lead crowning for the driven gear is from 7 μm to 21 μm. To simplify calculation, independent variables are coded to the usual (-1, 1) interval. Here, the coded variable x_1, x_2 and x_3 denote torque, lead crowning for the driving gear and lead crowning for the driven gear, respectively. Table 4 shows results of the peak-to-peak transmission error (y), calculated by transmission error program. Using the experimental results, the fitted regression model is given by

$$\hat{y} = 0.3762 - 0.1498x_1 + 0.132x_2 + 0.132x_3 + 0.1491x_1^2 - 0.0011x_2^2 \\ -0.0011x_3^2 - 0.1129x_1x_2 - 0.1129x_1x_3 + 0.006x_2x_3 \ . \tag{9}$$

Analysis of variance and coefficient of multiple determination are used to measure the usefulness of the model. Since F_o exceeds F (3,11;0.01) (Table 5), at least one of the regressor variables x_1, x_2, and x_3 contributes significantly to the model.

The coefficient of multiple determination R^2 is 0.9635. This means that the model explains approximately 96.35% of the observed variability.

Since the usefulness of the model is confirmed, the optimum problem can be defined as the minimization of Eq. (9), subject to $-1 \le x_1 \le 1, \quad -1 \le x_2 \le 1, \quad -1 \le x_3 \le 1$. Matlab is used for the solution of the multivariable constrained function [14, 15]. The solution has the local minima depending on the initial values. Many initial values, including initial value (0, 0, 0), produce the minimum transmission error 0.1469 at torque of 5.88 kgf·m, and lead crowning of 7 μm for the driving and driven gears.

4. Robust design of the tooth surface under multiple loads

The designed experiment can apply to the robust design. Robust design usually means to design or determine one or more of the following [16]:

1. "Designing systems (products or process) that are insensitive to environmental factors that can affect performance once the system is deployed in the field.
2. Designing products so that they are insensitive to variability transmitted by the components of the system.
3. Designing processes so that the manufactured product will be as close as possible to the desired target specifications even though some process variables or raw material characteristics are impossible to control precisely.
4. Determining the operating conditions for a process so that critical product characteristics are as close as possible to the desired target value and the variability around this target is minimized."

The robust parameter design introduced by G. Taguchi classifies the variables in a process or product as either controllable or uncontrollable (or noise) variables. The design also reduces product or process variation by choosing levels of controllable variables that minimize the variability transmitted to the response from the uncontrollable variables. Controllable variables in gears can be tip-relief and lead crowning, while uncontrollable variables can be multiple loads. In a sense, the design of the tooth surface can be the robust design.

Tip-relief and lead crowning are important modifications in the profile and lead curves, respectively. The tooth surface that consists of the profile and lead curves should have low transmission error under multiple loads. Tip-relief and lead crowning should be insensitive to multiple loads, and selected to robustly design the tooth surface under multiple loads. The region of exploration for fitting the second-order model is that torque is from 4 kgf·m to 10 kgf·m; tip relief and lead crowning for the driven gear is from 6 μm to 18 μm, and from 7 μm to 21 μm, respectively; 12 μm tip relief and 0 μm lead crowning for the driving gear. Table 6 shows the experimental results by transmission error program.

Using Table 6 results, regression models corresponding to torque 4, 7, and 10 kgf·m are built in a similar manner, as shown in Table 7. Here, the coded variables x_1 and x_2 denote the tip relief and lead crowning for the driven gear, respec-

Table 6. Experimental results for the robust design.

(a) Torque: 4 kgf·m

Run	x_1	x_2	y_1
1	-1	-1	0.1296
2	-1	0	0.2504
3	-1	1	0.4655
4	0	-1	0.1113
5	0	0	0.2972
6	0	1	0.5041
7	1	-1	0.1564
8	1	0	0.3218
9	1	1	0.5802

(b) Torque: 7 kgf·m

Run	x_1	x_2	y_2
1	-1	-1	0.1796
2	-1	0	0.1527
3	-1	1	0.2332
4	0	-1	0.2075
5	0	0	0.1096
6	0	1	0.2950
7	1	-1	0.2060
8	1	0	0.1168
9	1	1	0.3486

(c) Torque: 10 kgf·m

Run	x_1	x_2	y_3
1	-1	-1	0.2067
2	-1	0	0.3592
3	-1	1	0.2666
4	0	-1	0.2847
5	0	0	0.3395
6	0	1	0.1618
7	1	-1	0.3604
8	1	0	0.3346
9	1	1	0.1755

Table 7. Regression model corresponding to each torque.

Torque	Regression model	F_o/ P-value	R^2
4 kgf·m	$\hat{y}_1 = 0.2811 + 0.0355x_1 + 0.1921x_2 + 0.0131x_1^2 + 0.0347x_2^2 + 0.022x_1x_2$	1090.8/ 0	0.9973
7 kgf·m	$\hat{y}_2 = 0.125 + 0.0176x_1 + 0.0473x_2 + 0.0021x_1^2 + 0.1186x_2^2 + 0.0223x_1x_2$	31.18/ 0.0007	0.9122
10 kgf·m	$\hat{y}_3 = 0.3299 + 0.0063x_1 - 0.0413x_2 + 0.0218x_1^2 - 0.1018x_2^2 - 0.0612x_1x_2$	57.36/ 0.0001	0.9503

tively. Since F_o of each regression model exceeds F (2,6;0.01) =10.9, and the coefficient of multiple determination is more than 0.9122, the usefulness of the models is confirmed.

The lowest transmission error in a torque may not guarantee low transmission error in other torques. The design of the tooth profile for low noise should maintain low transmission error at all operating torques, although it is traded off at each torque. A weighted sum of the objective functions at 4, 7, and 10 kgf·m can satisfy the conditions, which leads to the multi-objective optimization problem as follows [17]:

$$\hat{y} = w_1\hat{y}_1 + w_2\hat{y}_2 + w_3\hat{y}_3 \, . \tag{10}$$

Because the objective functions have equal contribution in gears, the weighing functions w_1, w_2, w_3 have the constant of one-third. Equation (10) becomes

$$\begin{aligned}\hat{y} = (0.736 + 0.0594x_1 + 0.1981x_2 + 0.037x_1^2 \\ + 0.14312x_2^2 - 0.0169x_1x_2)/3\end{aligned} \, . \tag{11}$$

Therefore, robust design of the tooth surface under multiple loads results in the minimization problem of Eq. (11), subject to $-1 \le x_1 \le 1, -1 \le x_2 \le 1$. The constrained multi-variable func tion of Eq. (11) is solved through Matlab. The minimum of the constrained multivariable function is 0.211, obtained at the tip relief of 6 μm and lead crowning of 8.74 μm. The result of the minimum is validated in the graphic of Eq. (11), as shown in Fig. 4. The transmission error at the tip relief of 6 μm and lead crowning of 9 μm is 0.1549 at 4 kgf·m; 0.1522 at 7 kgf·m, and 0.2792 at 10 kgf·m. Considering the manufacturing tolerance, the profile and lead curves obtained in this analysis are shown in Fig. 5.

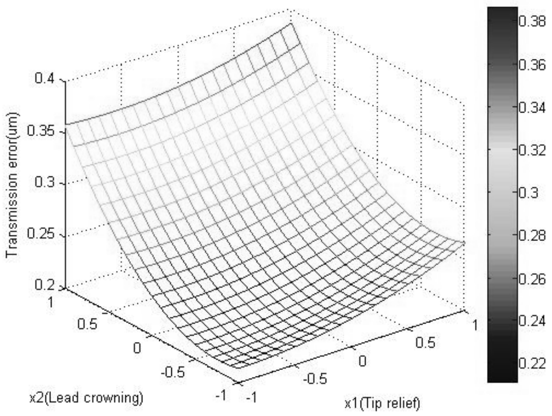
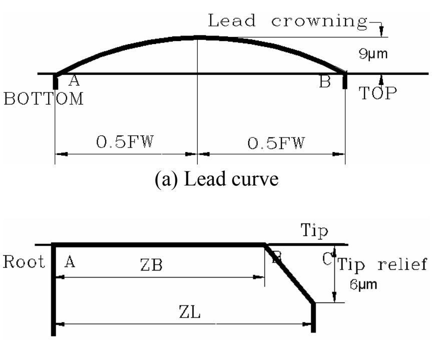


Fig. 4. Graphic of the objective function for the robust design.



(a) Lead curve

(b) Profile curve

Fig. 5. Profile and lead curve by the robust design.

5. Conclusions

This paper deals with the optimal and robust tooth surface for the reduction of helical gear noise. The simulation tool uses the developed program for analyzing transmission error correlated with gear noise. The method to determine the optimal lead curve, using the design of experiment and the response surface method, is proposed. The effect of transmission error on candidate design variables by a screening experiment is investigated using the analysis of variance. Design variables are likewise selected. The fitted regression model of transmission error is built with the statistic validation of the representation. The model with constraints is solved to obtain the optimum lead curve design. Finally, the method for the robust design of the tooth surface for low transmission error under multiple load conditions, using the fitted regression model and multi-objective optimization, is also proposed.

Acknowledgment

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Nomenclature

e (x)	:	Composite tooth errors
F	:	F-distribution
F_0	:	Test statistic
K_b	:	The influence function of bending deflection
K_c	:	The influence function of contact deformation
L	:	Contact line
p (x)	:	The load distribution of tooth
R_{b1}	:	Base radius of a driving gear
T	:	Input torque
W	:	Transmitted load
w_1, w_2, w_3	:	The weighing functions
x_1, x_2, x_3	:	The coded variables
Δ	:	Transmission error
β_b	:	Base helix angle
v_o	:	Contact deformation

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